

ST758, Homework 1

Due Thursday, Sep 11, 2014

Some R exercises

- Let $a = 0.7$, $b = 0.2$, and $c = 0.1$.
 - Test whether $(a + b) + c$ equals 1.
 - Test whether $a + (b + c)$ equals 1.
 - Test whether $(a + c) + b$ equals 1.
 - Explain what you found. Hint: find the internal representation of these numbers.
- Create the vector $\mathbf{v} = (969, 971, 972, \dots, 1022, 1023)$ of 54 elements.
 - Compute the sum $\sum_{i=1}^{54} 2^{v_i}$.
 - Compute the sum $\sum_{i=2}^{54} 2^{v_i}$.
 - Compute the sum $2^{v_1} + \sum_{i=2}^{54} 2^{v_i}$.
 - Explain what you found.
- Create the vector $\mathbf{x} = (0.988, 0.989, 0.990, \dots, 1.010, 1.011, 1.012)$.
 - Plot the polynomial $y = x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1$ at points x_i in \mathbf{x} .
 - Plot the polynomial $y = (x - 1)^7$ at points x_i in \mathbf{x} .
 - Explain what you found.
- Let $\mathbf{u} = (1, 2, 3, 3, 2, 1)^\top$.
 - Compute $\mathbf{U} = \mathbf{I} - (2/d)\mathbf{u}\mathbf{u}^\top$ where $d = \mathbf{u}^\top\mathbf{u}$. (This type of matrix is known as an ‘elementary reflector’ or a ‘Householder transformation.’)
 - Let $\mathbf{C} = \mathbf{U}\mathbf{U}$, the matrix product of \mathbf{U} and itself. Find the largest and smallest off-diagonal elements of \mathbf{C} .
 - Find the largest and smallest diagonal elements of \mathbf{C} .
 - Compute $\mathbf{U}\mathbf{u}$. (matrix times vector)
 - Compute the scalar $\max_i \sum_j |U(i, j)|$
 - Print the third row of \mathbf{U} .
 - Print the elements of the second column below the diagonal.
 - Let \mathbf{A} be the first three columns of \mathbf{U} . Compute $\mathbf{P} = \mathbf{A}\mathbf{A}^\top$.
 - Show that \mathbf{P} is idempotent by recomputing (e) with $\mathbf{P}\mathbf{P} - \mathbf{P}$.
 - Let \mathbf{B} be the last three columns of \mathbf{U} . Compute $\mathbf{Q} = \mathbf{B}\mathbf{B}^\top$.
 - Show that \mathbf{Q} is idempotent by recomputing (e) with $\mathbf{Q}\mathbf{Q} - \mathbf{Q}$.

- (l) Compute $\mathbf{P} + \mathbf{Q}$.
5. Read in the matrix in the file 'oringp.dat' on the failure of O-rings leading to the Challenger disaster. The columns are flight number, date, number of O-rings, number failed, and temperature at launch. Compute the correlation between number of failures and temperature at launch, deleting the last, missing observation (the disaster).
6. Let the $n \times n$ matrix \mathbf{A} have elements $A(i, j) = 1/(|i - j| + 1)$.
 - (a) Compute and print \mathbf{A} for $n = 10$.
 - (b) Compute and print the Cholesky factorization for \mathbf{A} for $n = 10$.
 - (c) Compute the Cholesky factorization for $n = 20$. Does it fail? If not, find the determinant.